Fluid-Solid Heat Transfer

- the nature of heat transfer -

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Introduction

This document should be seen as a guide to heat transfer between solid and fluids. It is under work…… This is a draft version…

Radiation Heat Transfer

All type of objects emits and absorbs thermal radiation (electromagnetic waves). Simply speaking, if a body is hotter than its surroundings it emits more radiation than it absorbs, and thus tends to cool down, and vice versa. Eventually thermodynamic equilibrium may be reached, a state where the rate of absorption equals the rate of emission. To describe the basic properties of thermal radiation the black body can be considered. A black body is an idealized body that absorbs all incident thermal radiation independent of wavelength, i.e. it represents a perfect absorber. A black body is also a perfect emitter, featuring a continuous frequency spectrum that only depends on the body's temperature (Planck's law) and diffuse emission, i.e. the radiation is emitted equally in all directions. A black body at room temperature emits radiation at the infrared portion of the spectrum and is thus not visible to the human eye. As the temperature of the body rises the frequency increases and the radiation becomes visible (at about 900 K), ranging from red, yellow-orange to bluish light with increasing temperature.

A real body does emit thermal radiation to a fraction of that for the ideal black body. This fraction is given by the emissivity, ε. The emissivity for a material surface depends on temperature, emission angle, and wavelength. The wavelength dependency is often not considered. This simplification is known as the gray body assumption. Often, for simplicity, also the dependency on the emission angle is neglected.

The emissivity for a certain solid surface may change over time due to a change in material properties, e.g. due to corrosion and ageing effects. An initially shiny and bright surface featuring low emissivity, like polished steel, may over time get more and more matte and darkish, causing the emissivity to increase. Hence, the rate of emitted radiation increases over time which may cause unexpected problems and severe damage in typical engineering applications. For example polished and oxidized steel have an emissivity of about 0.1 and 0.8, respectively.

Similar to the emissivity, the fraction of absorption relative to the ideal black body is given by the absorptivity, α. According to Kirchoff's law the absorptivity of a body is equal to its emissivity, i.e. $\alpha=\varepsilon$.

The total emissive power per unit surface area, summed over all wavelengths, is called the radiant intensity $I(T)$ [W/m^2], which is given by Stefan-Boltzmann law. Supplemented with the emissivity it can be written as:

$$I = \varepsilon\sigma T^4,$$

where σ is the Stefan-Boltzmann constant. As seen, the emitted power is less the more reflective a surface is, i.e. a perfectly reflective body is the so-called white body. It can also be concluded that contrary to convective heat transfer the amount of heat leaving the surface is independent of the material surrounding it.

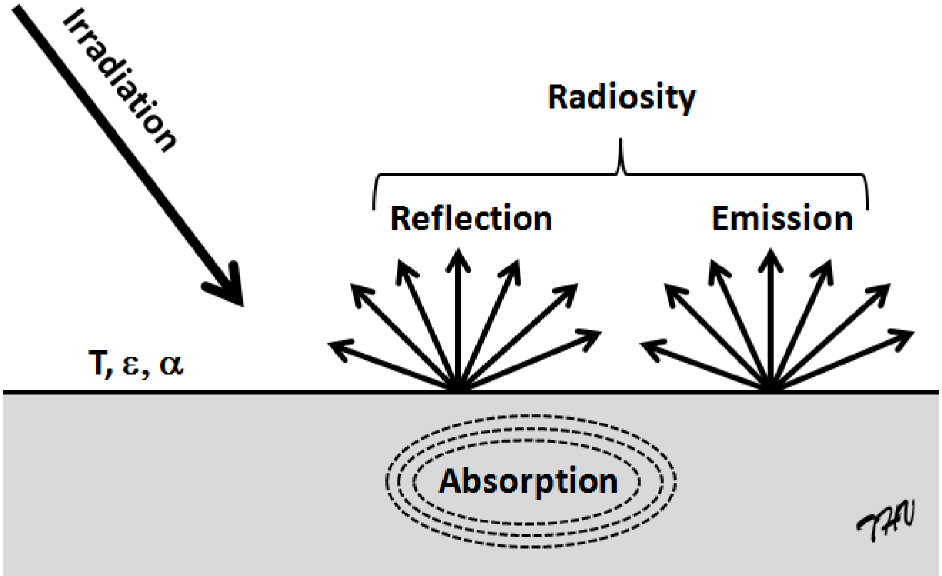

Figure XX depicts the basic mechanisms in thermal radiation. The incident radiation is called irradiation and is denoted *G*. The incident radiation is either absorbed or reflected by the body (transmission not being considered). The rate of reflected radiation is (1-ε)*G* and the rate of absorbed radiation is ε*G*. Similar to the emitted radiation, the reflected radiation is assumed to be diffuse, i.e. reflected in all possible directions. The net heat transfer at the surface is given by the difference between the incoming and total outgoing radiation. The total outgoing radiation is denoted radiosity, *J*, and can be expressed as:

$$J = \varepsilon\sigma T^4 + (1-\varepsilon)G$$

If two surfaces, A and B, surrounded by vacuum are being considered the radiosity of the first surface equals the irradiation of the other, assuming that all outgoing radiation from one surface hits the other. This corresponds to the, so-called, view factors $F_{A \to B}$ and $F_{B \to A}$ both being equal to unity. The view factor is the fraction of the total radiosity from a surface that strikes another surface.

Radiation between two idealized bodies, as in the computational study…..

In this study the radiosity and absorbtion at each solid surface will be analyzed and discussed for different emissivity and surface temperatures.

Convective Heat Transfer

Additional to radiation, heat transfer from or to a solid body can also be due to convective heat transfer. Convective heat transfer is the sum of diffusion heat transfer (molecular motion) and advective heat transfer (fluid bulk motion). Contrary to radiation these two mechanisms depend upon the character of the material surrounding the body. Whereas radiation can occur in vacuum, advection and diffusion cannot.

Heat transfer due to the motion of molecules is in fluids termed diffusion and in solids conduction heat transfer. This transport mechanism is driven by the temperature gradient in the material, contrary to radiation that is driven by the absolute value of temperature. In fluids diffusive mass transfer follow the same principles as diffusive heat transfer. Through random molecular motion (Brownian motion) particles tend to spread from regions featuring high concentration/temperature to regions featuring less concentration/temperature.

The ability for a material to transport heat by molecular motion is expressed by its conductivity, k [$W \cdot m^{-1} \cdot K^{-1}$]. In a material with low conductivity an initial non equilibrium state exist a longer time than in material with high conductivity. A common way to describe a materials willingness to transport heat by conduction is the thermal conductance, C, which is given by k/L [$W \cdot K^{-1} \cdot m^{-2}$]. It denotes the time rate of steady heat transfer through a unit area at 1 K temperature difference over a material with thickness L. The opposite to the thermal conductance is the thermal resistance, R-value, of a material. These measures will be further discussed in the next part of this document.

The rate of heat transfer, q [$W \cdot m^{-2}$], due to conduction is given by Fourier's law:

$$q_y = -k \frac{dT}{dy},$$

here seen in one-dimensional form. This equation clearly shows that the rate of heat transfer depends linearly to the conductivity and the temperature gradient. By dividing k with the density and the specific heat capacity the thermal diffusivity, α [$m^2 \cdot s^{-1}$], is obtained.

Contrary to diffusion, advective heat transfer is due to bulk motion (macroscopic motion) in the fluid. This motion can either be forced or natural. Natural advection originates from buoyancy effects as the density of a fluid is temperature dependent. In general when referring to heat transfer between a solid and a fluid the term convective heat transfer is used. This since both mechanisms are necessary in this process. Close to the surface of the solid the fluid motion is, due to strong viscous effects, small. Hence, diffusion is the dominant mechanism in heat transfer. At the surface, the wall-normal diffusion in the fluid equals the wall-normal conduction within the solid as there is no macroscopic fluid motion present. Further away from the surface the bulk motion is no longer negligible and advective heat transfer becomes the dominant transport mechanism. The convective heat transfer in a fluid is described by the advection-diffusion equation as:

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^{\ 2}}.$$

As seen in the above equation the right side term, *i.e.* the diffusion term, is the only transport term present close to the wall as the second term, i.e. the advection term, contains the velocity u_j, that tends to zero with decreasing wall distance. By normalization this equation can be rewritten as:

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This is helpful in determining the relative importance of the different transport mechanisms.

The rate at which heat is transferred from or to a solid surface, q_w , due to the motion of the adjacent fluid, depends on the wall-normal gradient of the fluid temperature, i.e. dT/dy, and the diffusivity of the fluid. The heat transfer between a solid and a fluid can be written as:

$$q_w = h(T_w - T_0),$$

where h is the heat transfer coefficient, a function of the flow, and T_0 the reference temperature. In the performed simulations the reference temperature is represented by the temperature at the channel inlet.

As known the heat transfer at the wall is solely due to conduction, why:

$$q_w = -k \left.\frac{dT}{dy}\right|_{y=0}.$$

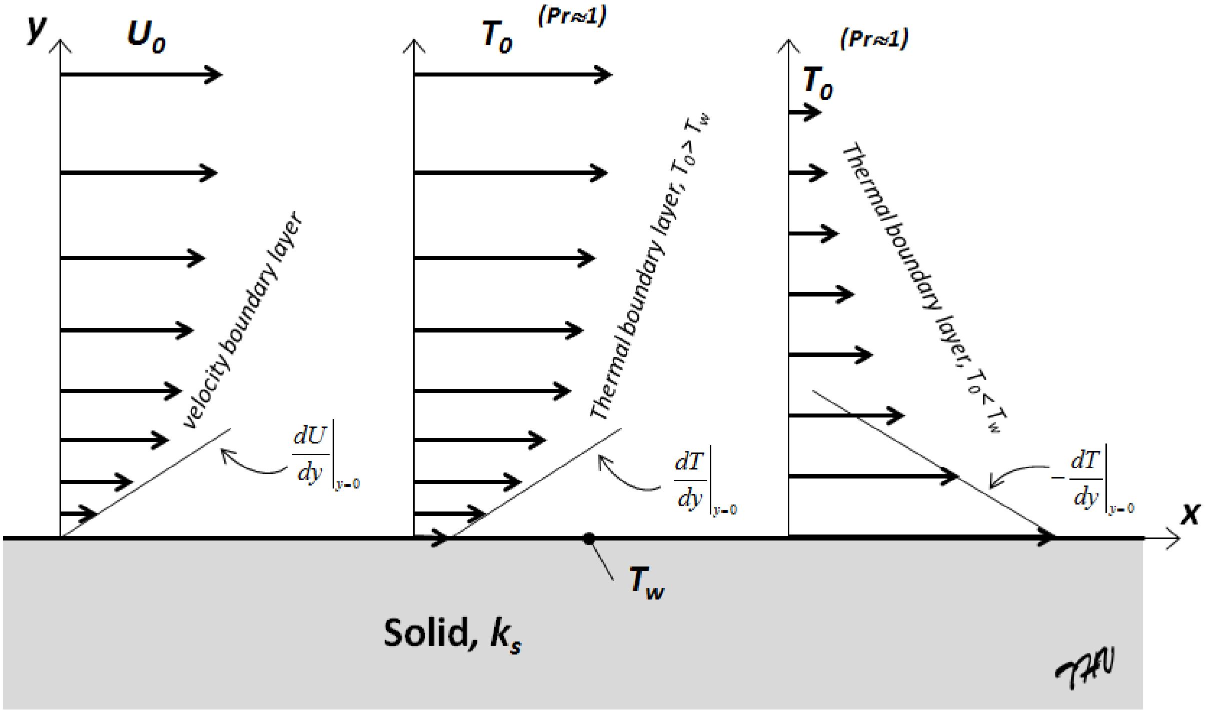
By combining these two expressions one can derive a dimensionless number called the Nusselt number, as:

$$Nu = \frac{hL}{k} = -\frac{L}{T_w - T_0} \left.\frac{dT}{dy}\right|_{y=0},$$

where L is a characteristic length scale. The Nusselt number expresses the ratio of convective heat transfer to conductive heat transfer between a solid and a moving fluid. Under specific conditions the heat and momentum transfer can be related according to Reynolds analogy as:

$$Nu \propto C_f \operatorname{Re} \operatorname{Pr},$$

where C_f is the wall friction coefficient, *Re* the Reynolds number and *Pr* the Prandtl number (defines the ratio between viscous diffusion to thermal diffusion). Obviously, the Nusselt number is proportional to the skin friction. The skin friction itself is proportional to the wall normal gradient of the fluid velocity, i.e. the behavior of the temperature boundary layer is similar to that of the velocity boundary layer (assuming $Pr \approx 1$). Hence, good understanding of the velocity boundary layer is necessary for good understanding of the wall heat transfer.



Consider the simple channel used in this study. The height of one channel is *H*. The mean velocity of the fluid at the inlet is *U* and the kinematic viscosity of the fluid is ν. From these three numbers a Reynolds number can be defined as:

$$\mathrm{Re}_H = \frac{UH}{\nu}.$$

The Reynolds number is dimensionless and defines the ratio of inertial forces to viscous forces. At low Reynolds numbers the flow features strong viscous forces and is hence typically stable. At high Reynolds number stabilizing viscous effects are negligible and the flow becomes unstable, and eventually turbulent. If the Reynolds number, Re_H, is sufficiently large and kept fixed the wall friction may differ depending on what state the flow is in. The state of the flow can in this case be defined by a different Reynolds number, say Re_L, where *L* denotes the distance from the inlet of the channel. For small *L* the flow is laminar, i.e. stable. The strong viscous effects cause the velocity boundary layer to be thick as the molecular motion tends, similar as for conduction, to even out strong gradients. As the velocity boundary layer is thick so also is the temperature boundary layer. This results, in turn, in a weak temperature gradient, and thus in low wall heat transfer rates.

Further downstream the channel Re_L becomes eventually high enough to turn the flow into turbulent flow. The turbulent flow is categorized by great mixing capabilities as it is highly diffusive. The flow consists of a wide range of so-called eddies (swirling structures) of different length and velocity scales. The turbulent energy goes from the largest down to the smallest eddies, a process known as the energy cascade. The turbulent energy is eventually dissipated by viscous forces, i.e. turbulent flow is dissipative. Contrary to the laminar flow the fluid velocity close to the wall is relatively high in a turbulent flow. This is due to the motion of the energetic eddies that feed the near-wall region with momentum from the outer regions. In the same manner, recall the great mixing capabilities, it feeds the near-wall region with fresh cold fluid from the outer regions (considering a hot solid and a cold fluid). Hence, the velocity boundary layer becomes steep, and so also the temperature boundary layer. The wall heat transfer rate is, thus, generally high for turbulent flows.

Heat Transfer Correlations

There are a large number of different empirical correlations that express the rate of convective heat transfer for different flow and geometrical conditions. One example is the Dittus-Boelter correlation that is valid for strict forced convection cases featuring fully turbulent pipe flow. The heat transfer coefficient can be expressed as:

$$h = Nu\frac{k}{D} = \left(0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}\right)\frac{k}{D}$$

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Correlation for flat plate laminar and turbulent flows……………..

Combining Radiation and Convective Heat Transfer

If the thermal radiation is not, in any way, directly influenced by the presence of a fluid, i.e. similar behavior as in vacuum, the only influence from the fluid flow on the radiation is indirect via a change in surface temperature. In the same manner, the convection heat transfer is only influenced by the presence of thermal radiation by a change in surface temperature.

Usually, the system is said to be in thermal equilibrium when there is no exchange of heat energy between the different materials, i.e. they are at the same temperature. For the present system this can only occur when the fluid is not allowed to leave the domain of interest and there is no applied boundary conditions preventing the system from reaching a spatially uniform temperature. This is so since the initial temperature of the fluid deviate from that of the heat source. However, by some authors the system is said to be in equilibrium when there is no change of temperature over time. If this definition is considered all performed studies herein are in thermal equilibrium.

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Computational Models

As in most type of computations involving some type of modeling specific approximations are made. Regarding the radiation modeling the fluid is assumed to not absorb nor reflect radiation (S2S-model). The radiation is assumed to be reflected and emitted in a diffuse manner at walls (Gray Thermal Radiation model). Furthermore, in the calculation of view factors a finite number of rays are considered. However, for the cases considered herein the number of rays is more than enough as the geometry is simple.

Regarding the treatment of the fluid flow standard methods and settings have been considered. The flow is computed by the Reynolds Averaged Navier Stokes approach. The turbulence model used is the Realizable k-ε model. The fluid, which here is represented by air, is assumed to be an ideal gas. For more information about computation of fluid flows see REFREFREF.